ANALYTICAL MODEL FOR PREDICTING BRITTLE FAILURES OF BOLTED ULTRA-HIGH PERFORMANCE CONCRETE JOINTS

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1. Introduction

Ultra-high performance concrete (UHPC) is a cementitious composite material with exceptional mechanical and durability properties, including sustained postcracking tensile strength, a remarkable self-compacting ability and a high-compressive strength. Laboratory tests of structural elements have clearly indicated that UHPC components can exhibit compressive and tensile mechanical properties higher of those expected from conventional and fiber-reinforced concretes. Currently, UHPC precast construction is regarded as an interesting alternative to be considered in civil engineering projects to take advantage of its exceptional material properties. Discovering a practical and reliable method of connecting ultra-high performance concrete precast elements with fasteners has been recognised as a critical factor (*Maya et al. Experimental assessment of connections for precast concrete frames using ultra high performance fibre reinforced concrete, Construction and Building Materials 2013*). Unfortunately, the research in bolted UHPC joints is so far insufficient (*Camacho et al. UHPFRC bolted joints: failure modes of a new simple connection system. High Performance Fiber Reinforced Cement Composites RILEM 2012*).

The objective of this research is to present a comprehensive analytical method capable of predicting the ultimate strength of bolted ultra-high performance concrete joints. The analytical solution uses the stress functions expressed in terms of complex parameters, and it can be considered an application of Lekhnitskii's theory on stress distributions in anisotropic plates.

2. Development of basic equations

The stress distribution around a pin-loaded hole in an elastically orthotropic or isotropic plate is investigated for the central section of a double-lap mechanical joint (see Fig. 1).

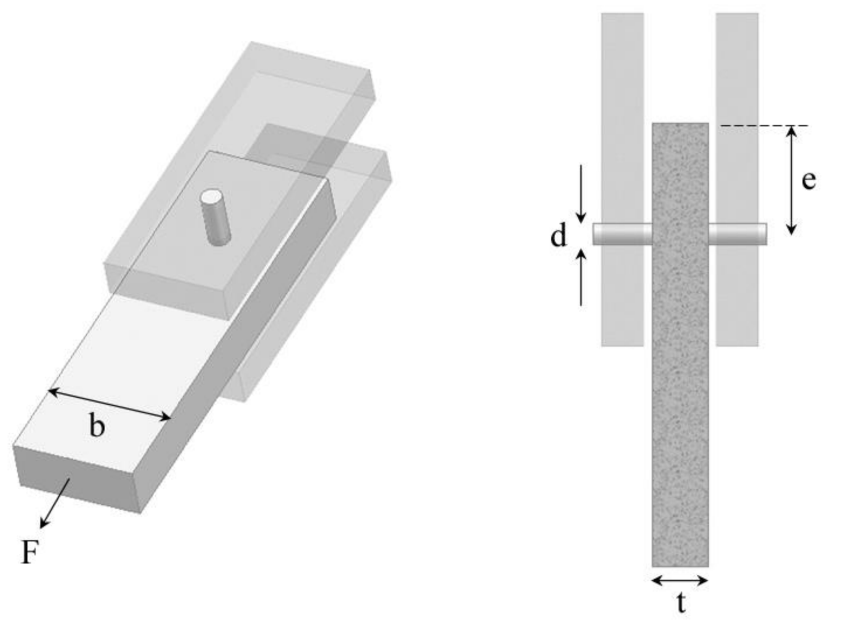

Figure 1: Double-lap mechanical joint.

Making use of the symmetry of the specimen (since the principal material and geometric axes are coincident), the final geometry analyzed is shown in Fig. 2.

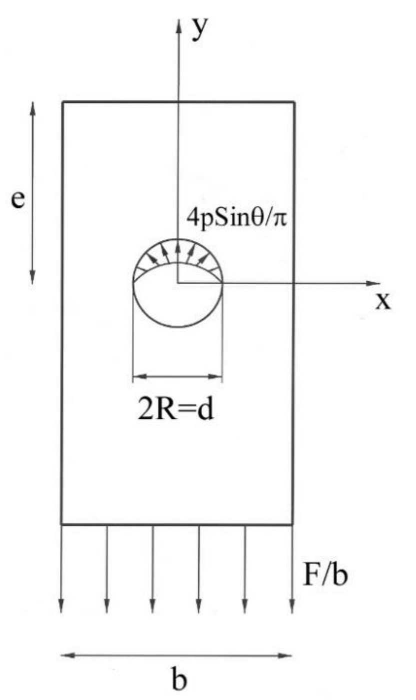

Figure 2: Geometry for the joint under a sinusoidal distribution of load.

The hole is loaded frictionless on only a part of its edge by an infinitely rigid pin which has the same diameter as the hole. The loading force is distributed along the edge of the hole in a shape represented by ***4pSinθ/π*** (see Fig. 2) then the loading force ***F*** resulting from this symmetrical distribution of load related to the axis ***y*** is of magnitude ***2pRt***, where ***p*** is the bearing stress according to the classical definition, ***R*** is the radius of the hole and ***t*** is the unit thickness of the plate.

With the assistance of the equations below, it is possible to estimate the stresses in any point of the orthotropic joint. For plane stress situations in orthotropic plates the stresses can be expressed by means of derivatives of two stress complex functions ***φ(z₁)*** and ***ψ(z₂)***, which are arbitrary functions of the complex variables z_1 and z_2.

$$\sigma_x = 2\,\mathbf{Re}\left\{\mu_1^2\varphi'(z_1)+\mu_2^2\Psi'(z_2)\right\}$$

$$\sigma_y = 2\,\mathbf{Re}\left\{\varphi'(z_1)+\Psi'(z_2)\right\}$$

$$\tau_{xy} = -2\,\mathbf{Re}\left\{\mu_1\varphi'(z_1)+\mu_2\Psi'(z_2)\right\}$$

In general, the stress complex functions can be estimated from the boundary conditions of the problem. The pertinent solutions for the pin-loaded hole in an elastically orthotropic or isotropic plate problem have been obtained by Echavarría (*Echavarría C. Analyse d'une plaque orthotrope avec trou: Application aux assemblages en bois. PhD thesis N° 2947, Swiss Federal Institute of Technology Lausanne EPFL, Switzerland, 2004*) and will be summarized here.

The perpendicular stress $\boldsymbol{\sigma_x}$ and the longitudinal stress $\boldsymbol{\sigma_y}$ at the point with coordinates (*x = 0 , y = R*) can be expressed as follows:

$$\sigma_x = \left\{\frac{(4+\pi)Fn\omega}{2R\pi^2}\right\} - \left(\frac{Fk}{2b}+\frac{3Fk}{2R\pi}\right) - \frac{\upsilon_{xy}F}{2R\pi} \qquad \sigma_y = -\left(\frac{2F}{R\pi}\right)$$

Now, at the point with coordinates (*x = R , y = 0*) , the stresses are:

$$\sigma_y = \left(\frac{F}{2b}+\frac{2F}{R\pi^2}\right)\frac{n}{k}+\frac{F}{2b} \qquad \sigma_x = 0$$

3. Predicting brittle failures of a bolted ultra-high performance concrete joint

There are four common failure modes in bolted joints made of UHPC plates, namely tension, shear-out, bearing and cleavage. Unfortunately, brittle failures in UHPC bolted joints with insufficient end distance, edge distance, and (or) fastener spacing occur predominantly due to tension. In this research, a number of example problems have been solved using the analytical formulation described above for a particular UHPC joint. Two parameters were investigated: the first involved the ***b/d*** ratio, and the second was the ***e/d*** ratio. For clarity, the calculated stresses are normalized by the average bearing stress ***p = F/d*** and the tensile stresses are reported as positive. Let's consider, for instance, the stress distributions calculated for a UHPC element of a unit thickness ***t*** and width ***b*** assuming the elastic properties of UHPC shown in Table 1.

Table 1: Elastic constants of UHPC

E_x (GPa)	E_y (GPa)	G_{xy} (GPa)	ν_{yx}
45	45	18.75	0.20

In Table 2, the stress-concentration factors at the point with the highest perpendicular stress $\boldsymbol{\sigma_x}$, at (*x = 0 , y = R*) , are given for four ratios ***b/d*** and ***e/d***. Similarly, results obtained for highest longitudinal stress $\boldsymbol{\sigma_y}$, at (*x = R , y = 0*) are listed in Table 3.

Table 2: Highest perpendicular stress concentration σ_x, at the point (*x = 0 , y = R*)

e/d	b/d = ∞	b/d = 16	b/d = 8	b/d = 4
2	0.43*p*	0.40*p*	0.37*p*	0.30*p*
4	-0.46*p*	-0.49*p*	-0.53*p*	-0.59*p*
8	-0.71*p*	-0.74*p*	-0.77*p*	-0.84*p*
16	-1.02*p*	-1.05*p*	-1.08*p*	-1.14*p*

Table 3: Highest longitudinal stress concentration σ_y, at the point (*x = 0 , y = R*)

e/d	b/d = ∞	b/d = 16	b/d = 8	b/d = 4
2, 4, 8, 16	0.81*p*	0.90*p*	1.00*p*	1.19*p*

4. Conclusions

From the analytical results presented here, it can be concluded that the diminution of the ***b/d*** ratio will cause an increase in the longitudinal stress concentration and, at the same time, will cause a diminution in the perpendicular stress concentration. The magnitude of the perpendicular stress concentration increases drastically when the end distance is less than ***4d***. Assuming brittle performance of UHPC in tension, the joint under consideration is assumed to break as soon as the state of stress at some point fulfils the failure criteria. In this research, the maximum stress criteria are used.

- It is clear that the highest longitudinal tensile stress $\boldsymbol{\sigma_y}$, which is independent of ***e/d***, increases when ***b/d*** decreases. For small values of ***b/d***, the highest longitudinal tensile stress will cause a pure tension failure in materials with low ultimate longitudinal tensile strength (i.e. UHPC joints, see Fig. 3).

Figure 3: Pure tension failure in UHPC joints.

- The highest perpendicular tensile stress $\boldsymbol{\sigma_x}$, will cause a splitting type of failure of the joint in materials with a low ultimate perpendicular tensile strength (i.e. wood joints).
- The high longitudinal compressive stress $\boldsymbol{\sigma_y}$, which is independent of ***e/d*** and ***b/d***, may lead to the bearing type of failure (i.e. steel joints).

The solution proposed here is entirely analytical and the most important results are compiled in a clear form of simple formulae. The results obtained provide useful information for design of bolted ultra-high performance concrete joints. Most importantly, the established model is the first one to provide an analytical basis for brittle failure modeling of ultra-high performance concrete joints.